Lecture 4

$\bar{F} = (xz^2 - xy^2)\vec{i} + (x^2y - yz^2)\vec{j} + (y^2z - x^2z)\vec{k}$

prove it's solinoidal.

$\bar{\nabla}\cdot\bar{F} = (z^2 - y^2) + (x^2 - z^2) + (y^2 - \cancel{x}^2)$

$= 0$

It's Solinoidal

$\phi = xyz^2$ find it's grad and prove That curl for grad of grad is zero

$\bar{\nabla}\phi = yz^2\vec{i} + xz^2\vec{j} + 2xyz\vec{k}$

$$\bar{\nabla}\times\bar{\nabla}\phi = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & xz^2 & 2xyz \end{vmatrix}$$

$= (2xz - 2xz)\vec{i} - (2yz - 2yz)j + (z^2 - z^2)k$

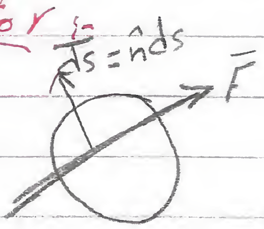
$= \vec{0}$

Flux of F vector :-

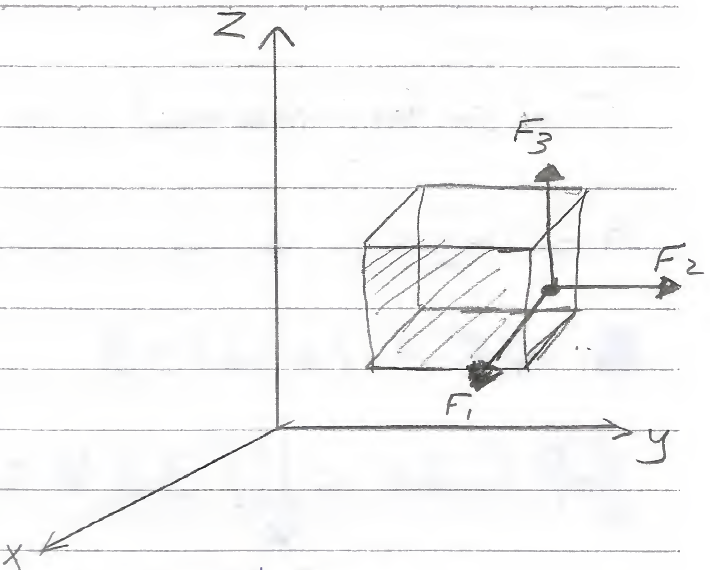
فيض الاتجاه

$\text{Flux} = \bar{F}\cdot\bar{ds}$

$= \bar{F}\cdot\hat{n}\,ds$



$\oint\!\!\oint \bar{F} \cdot d\bar{s}$



- Field passing through blue colored face

$$= \left(F_1 + \frac{\partial F_1}{\partial x} \cdot \frac{dx}{2}\right) dy\,dz$$ مساحة الوجه

- Field passing through the opposing face

$$= \left(F_1 + \frac{\partial F_1}{\partial x} \cdot -\frac{dx}{2}\right) dy\,dz$$

Net flux of both ; $\frac{\partial F_1}{\partial x} \cdot dx\,dy\,dz$

احنا هنا بنتكلم عن الـ (flux) الى هو الفيض ولو جينا اى جسم منتظم هيكون الـ (flux) الى هيسقط على مساحة سطحه من كل الى هتكمل فالجسم كله (حجمه).

Total Flux through all faces of vector $\bar{F}$

$$\iiint_V \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dx\,dy\,dz$$

Gauss Law : $\oint\!\!\oint_S \bar{F} \cdot d\bar{s} = \iiint \bar{\nabla} \cdot \bar{F}\, dV$

$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$

Find surface integral and evaluate its surface integral if it's متوازى مستطيلات with These الأحرف

$\vec{a} = \vec{i} + \vec{j} + \vec{k}$ ، $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$ ، $\vec{c} = \vec{i} - \vec{j} + \vec{k}$

$\bar{\nabla} \cdot F = 1 + 1 + 1 = 3$

$$\oiint_S \bar{F} \cdot ds = \iiint_V 3\, dV = 3 \iiint_V dv = 3V$$

"Remember"

Volume of متوازى المستطيلات $= \bar{a} \cdot \bar{b} \times \bar{c}$

$$3V = 3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 3(0) = 0$$

If $\bar{H} = \bar{\nabla}_x \bar{A}$ prove that any closed surface is $\oiint F \cdot d\bar{s} = 0$

$$\because \oiint \bar{F} \cdot d\bar{s} = \iiint_V \bar{\nabla} \cdot \bar{F}\, dv$$

$$\iiint_V (\bar{\nabla} \cdot \bar{\nabla}_x A)\, dV = 0$$

$$\oiint_S \bar{H} \cdot d\bar{s} = 0$$

$\bar{\nabla} \cdot \bar{\nabla}_x A = \text{zero}$ ← مثبتة من قبل

Stokes theorem:

$$\iint_S \bar{\nabla}_\times \bar{F} \cdot d\bar{s} = \oint_c \bar{F} \cdot d\bar{r}$$

Green theorem:

$$\iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\, dy = \oint_c (F_1 \bar{i} + F_2 \vec{j}) \cdot (dx \vec{i} + dy \vec{j})$$